Question Paper-Delhi (2012)

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three Sections A, B and C, Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION-A

Questions numbers 1 to 10 carry 1 mark each.

Q1. If a line has direction ratios 2, –1, –2, then what are its direction cosines? **1**

Q2. Find 'λ' when the projection of $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units. **1**

Q3. Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$. **1**

Q4. Evaluate : $\int_2^3 \frac{1}{x} dx$ **1**

Q5. Evaluate $\int (1-x)\sqrt{x}\, dx$. **1**

Q6. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}. **1**

Q7. If $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$, write the value of x. **1**

Q8. Simplify : $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ **1**

Q9. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$. **1**

Q10. Let * be a 'binary' operation on N given by a * b = LCM (a, b) for all $a, b \in$ N. Find 5 * 7. **1**

SECTION-B

Question numbers 11 to 22 carry 4 mark each.

Q11. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$. **4**

OR

If $\sin y = x \sin (a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

Q12. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%? **4**

Q13. Find the Vector and Cartesian equations of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. **4**

Q14. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12$ and $|\vec{c}| = 13$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}\,.\,\vec{b} + \vec{b}\,.\,\vec{c} + \vec{c}\,.\,\vec{a}$. **4**

Q15. Solve the following differential equation : **4**

$$2x^2 \frac{dy}{dx} - 2xy + y^2 = 0.$$

Q16. Find the particular solution of the following differential equation. **4**

$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ where $x = 0$.

Q17. Evluate : $\int \sin x \sin 2x \sin 3x \, dx$

OR

Evaluate : $\int \frac{2}{(1-x)(1+x^2)} dx$

Q18. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$.

OR

Using differentials, find the approximate value of $\sqrt{49.5}$. **4**

Q19. If $y = (\tan^{-1} x)^2$, show that **4**

$$(x^2+1)^2 \frac{d^2y}{dx^2} + 2x(x^2+1)\frac{dy}{dx} = 2.$$

Q20. Using properties of determinants, prove that **6**

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Q21. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}, x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. **6**

OR

Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.

Q22. Let A = R – {3} and B = R – {1}. Consider the function $f : A \rightarrow B$ defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Show that f is one-one and onto and hence find f^{-1}. **6**

SECTION-C

Questions numbers 23 to 29 carry 6 mark each.

Q23. Find the equation of the plane determined by the points A(3, –1, 2), B (5, 2, 4) and C(–1, –1, 6) and hence find the distance between the plane and the point P(6, 5, 9). **6**

Q24. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain 'A' grade and 20% of day scholars attain 'A' grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an 'A' grade, what is the probability that the student is a hostlier? **6**

Q25. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ` 17.50 per package on nuts and ` 7 per package of bolts. How many packages of each should be produced each day so as to maximize his profits if he operates his machines for at the most 12 hours a day? Form the above as a linear programming problem and solve it graphically. **6**

Q26. Prove that $\int_0^{\pi/4}(\sqrt{\tan x}+\sqrt{\cot x})dx=\sqrt{2}.\frac{\pi}{2}$ **6**

OR

Evaluate $\int_1^3(2x^2+5x)dx$ as a limit of a sum.

Q27. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$. **6**

Q28. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base. **6**

Q29. Using matrices, solve the following system of linear equations :

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

OR

Using elementary operations, find the inverse of the following matrix :

$$\begin{pmatrix}-1 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{pmatrix}$$

Marking Scheme

Class-XII

Mathematics (March 2012)

Q.No.	Value Pooints/Solution	65/1//1	Marks.

SECTION-A

1-10

1. $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$ **2.** $\lambda = 5$ **3.** $-4\hat{j} - \hat{k}$ **4.** $\log\left(\frac{3}{2}\right)$ **5.** $\frac{3}{2}x^{3/2} - \frac{2}{5}x^{5/2} + c$

6. $M_{2,3} = 7$ **7.** 13 **8.** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **9.** $\frac{2\pi}{3}$ **10.** 35. $1 \times 10 = 10$

SECTION-B

11.

$(\cos x)^y = (\cos y)^x \Rightarrow y \log \cos x = x \log \cos y$ 1/2

$\therefore \; y.\frac{(-\sin x)}{\cos x} + \log \cos x.\frac{dy}{dx} = x\frac{(-\sin y)}{\cos y}\frac{dy}{dx} + \log \cos y$ 1+1

$(\log \cos x + x \tan y)\frac{dy}{dx} = \log \cos y + y \tan x$ 1

$\therefore \; \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$ 1/2

OR

$\sin y = x \sin(a+y) \Rightarrow \cos y \frac{dy}{dx} = x \cos(a+y)\frac{dy}{dx} + \sin(a+y)$ 1

$\therefore \; \frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)}$ 1

$x = \frac{\sin y}{\sin(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - \frac{\sin y}{\sin(a+y)}.\cos(a+y)}$ 1

$\therefore \; \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y)\cos y - \cos(a+y)\sin y} = \frac{\sin^2(a+y)}{\sin a}$ 1

12. Let the coin be tossed n times

$\therefore$ P(getting at least one heat) $> \frac{80}{100}$ 1

$\therefore \; 1-P(0) > \frac{8}{10} \Rightarrow P(o) < 1-\frac{8}{10} = \frac{2}{10} = \frac{1}{5}$ 1

$\therefore \; {}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n < \frac{1}{5}$ or $\frac{1}{2^n} < \frac{1}{5}$ or $2^n > 5$ 1

$\Rightarrow n = 3.$ 1

13. Let the vector equation of required line be $\vec{a} = \vec{a} + \lambda\vec{b}$

than $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

and $\vec{b} = (3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})$ 1

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$ 1

$\therefore$ Vector equation of line is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k})$

or $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ } 1

and cartesian from is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ 1

14. $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$ 1/2

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ 1

or $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ 1

$\therefore \; \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{1}{2}(25 + 144 + 169) = -169.$ 1

15. $2x^2\frac{dy}{dx} - 2xy + y^2 = 0 \Rightarrow \frac{dy}{dx} = \frac{2xy - y^2}{2x^2} = \frac{2\frac{y}{x} - \frac{y^2}{x^2}}{2}$ 1/2

Putting $\frac{y}{x} = v$ so that $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\therefore \; v + x\frac{dv}{dx} = v - \frac{1}{2}v^2 \;\; \therefore \; x\frac{dv}{dx} = -\frac{1}{2}v^2$ 1/2

$\Rightarrow 2\int\frac{dv}{v^2} = -\int\frac{dx}{x} \Rightarrow \frac{2}{v} = \log x + c$ 1

$\therefore \; \frac{2x}{y} = \log x + c \;\; \therefore \; y = \frac{2x}{\log x + c}$ 1

16. $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2 = (1+x^2)(1+y^2)$ ½

$\Rightarrow \int \frac{dy}{1+y^2} = \int (1+x^2)dx$ 1

$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c$ 1

$x = 0, y = 1 \Rightarrow c = \pi/4$ 1

$\therefore \tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$ or $y = \tan\left(\frac{\pi}{4} + x + \frac{x^3}{3}\right)$ ½

17. $I = \int \sin x \sin 2x \sin 3x dx = \frac{1}{2}\int 2\sin 3x \sin x . \sin 2x dx$ ½

$= \frac{1}{2}\int (\cos 2x - \cos 4x)\sin 2x dx = \frac{1}{2}\int (\sin 2x \cos 2x - \cos 4x \sin 2x)dx$ ½

$= \frac{1}{4}\int \sin 4x dx - \frac{1}{4}\int 2\cos 4x \sin 2x dx$ 1

$= -\frac{1}{16}\cos 4x - \frac{1}{4}\int (\sin 6x - \sin 2x)dx$ 1

$= -\frac{1}{16}\cos 4x + \frac{1}{24}\cos 6x - \frac{1}{8}\cos 2x + c$ 1

OR

$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ ½

$2 = A(1+x^2) + (Bx+C)(1-x)$ 1½

$\Rightarrow 0 = A - B, B - C = 0 \quad A + C = 2 \quad \Rightarrow A = B = C = 1$

$\therefore \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x+1}{x^2+1}dx$ ½

$= -\log|1-x| + \frac{1}{2}(x^2+1) + \tan^{-1} x + c$ 1½

18. Slope of tangent, $y = x - 11$ is 1 ½

$y = x^3 - 11x + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ ½

If the point is (x_1, y_1) then $3x_1^2 - 11 = 1 \Rightarrow x_1 = \pm 2$ 1

$x_1 = 2$ then $y_1 = 8 - 22 + 5 = -9$ and if $x_1 = -2$ then $y_1 = 19$ 1

Since (–2, 19) do not lie on the tangent $y = x - 11$ ½

$\therefore$ Required point is (2, – 9) ½

OR

Let $y = \sqrt{x} \quad \therefore \quad y + \Delta y = \sqrt{x + \Delta x}$ ½

$$\Rightarrow \; y + \frac{dy}{dx}\Delta x \approx \sqrt{x + \Delta x}$$

$$\Rightarrow \; \sqrt{x} + \frac{1}{2\sqrt{x}}.\Delta x \approx \sqrt{x + \Delta x}$$ 1

Putting $x = 49$ and $\Delta x = 0.5$ we get 1

$$\sqrt{49} + \frac{1}{2\sqrt{49}}(0.5) \approx \sqrt{49.5}$$ ½

$$\Rightarrow \; \sqrt{49.5} = 7 + \frac{1}{28} = 7.0357$$ 1

19. $y = (\tan^{-1} x)^2 \; \Rightarrow \; \dfrac{dy}{dx} = 2\tan^{-1} x \dfrac{1}{1+x^2}$

$$\Rightarrow \; (1+x^2)\frac{dy}{dx} = 2\tan^{-1} x$$

$$\therefore \; (1+x^2)\frac{d^2y}{dx^2} + 2x.\frac{dy}{dx} = \frac{2}{1+x^2}$$

$$\Rightarrow \; (1+x^2)^2.\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2.$$

20. Using $R_1 \to R_1 + R_2 + R_3$ we get

$$\text{LHS} = \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$ 1

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$ 1

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$ Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ 1

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{RHS}$$ Using $R_1 \to R_1 + R_2 + R_3$, $R_2 \to -R_2$, $R_3 \to -R_3$ 1

21. $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right)$ 1

$=\tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$ 1+1

$=\frac{\pi}{4}-\frac{x}{2}$ 1

OR

Writing $\sin^{-1}\left(\frac{8}{17}\right)=\tan^{-1}\frac{8}{15}$ and $\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\frac{3}{4}$ 1

$\therefore$ LHS $=\tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4}=\tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15},\frac{3}{4}}\right)=\tan^{-1}\left(\frac{77}{36}\right)$ 1+1

Getting $\tan^{-1}\left(\frac{77}{36}\right)=\cos^{-1}\left(\frac{36}{85}\right)$ 1

22. Let $x_1, x_2 \in \text{A}$ and $f(x_1)=f(x_2)$ ½

$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3} \quad \therefore \quad x_1x_2-2x_2-3x_1=x_1x_2-2x_1-3x_2$

$\Rightarrow x_1 = x_2$ 1

Hence f is 1 – 1

Let $y \in \text{B}, \;\therefore\; y=f(x)\Rightarrow y=\frac{x-2}{x-3}\Rightarrow xy-3y=x-2$

or $x=\frac{3y-2}{y-1}$ ½

Since $y \neq 1$ and $\frac{3y-2}{y-1}\neq 3 \;\therefore\; x\in \text{A}$

Hence f is ONTO 1

and $f^{-1}(y)=\frac{3y-2}{y-1}$ 1

SECTION-C

23. Normal to the plane is $\vec{n}=\overrightarrow{\text{AB}}\times\overrightarrow{\text{BC}}$ ½

$\therefore \quad n=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 2\\ -6 & -3 & 2\end{vmatrix}=12\hat{i}-16\hat{j}+12\hat{k}$ 1½

∴ Equation of plane is

$$\vec{r}.(12\hat{i}-16\hat{j}+12\hat{k}) = (3\hat{i}-\hat{j}+2\hat{k}).(12\hat{i}-16\hat{j}+12\hat{k})$$
$$= 76$$ 2

or $\vec{r}.(3\hat{i}-4\hat{j}+3\hat{k}) = 19$ or $3x-4y+3z-19=0$

Distance of plane from the point P(6, 5, 9) is

$$d = \frac{|18-20+27-19|}{\sqrt{9+16+9}} = \frac{6}{\sqrt{34}}$$ 2

24. Let E_1 : selected student is a hostlier

E_2 : selected student is a day scholar 1

A : selected student attain 'A' grade in exam.

$$P(E_1) = \frac{60}{100}, \quad P(E_2) = \frac{40}{100}$$ 1

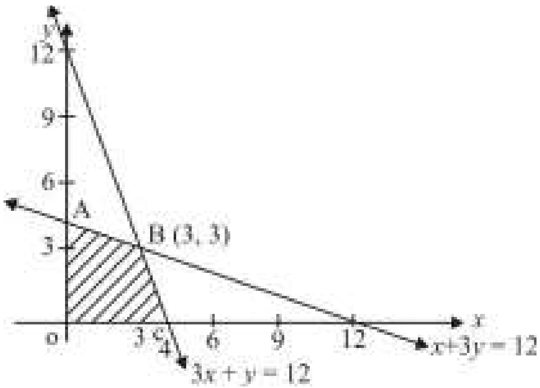
$$P(A/E_1) = \frac{30}{100}, \quad P(A/E_2) = \frac{20}{100}$$ 1

$$P(E_1/A) = \frac{P(E_1) \,.\, P(A/E_1)}{P(E_1) \,.\, P(A/E_1) + P(E_2) \,.\, P(A/E_2)}$$ 1

$$= \frac{\frac{60}{100}.\frac{30}{100}}{\frac{60}{100}.\frac{30}{100}+\frac{40}{100}.\frac{20}{100}} = \frac{9}{13}$$ 1+1

25. Let x package of nuts and y package of bolts be produced each day

∴ LPP is maximise $P = 17.5x + 7y$ 1



subject to $x + 3y \le 12$

$3x + y \le 12$ 2

$x \ge 0, y \ge 0$ correct graph

vertices of feasible region are A(0, 4), B (3, 3), C (4, 0)

Profit is maximum at B(3, 3)

i.e. 3 package of nuts and 3 package of bolts 1

26. $I = \int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})dx = \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$ 1

Putting $\sin x - \cos x = t$, to get $(\cos x + \sin x)dx = dt$ 1

and $\sin x \cos x = \frac{1-t^2}{2}$ 1

$\therefore \quad I = \sqrt{2}\int_{-1}^{0} \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}.[\sin^{-1} t]_{-1}^{0}$ 1+1

$= \sqrt{2}(\sin^{-1} 0 - \sin^{-1}(-1) = \sqrt{2}.\frac{\pi}{2}$ 1

OR

$I = \int_1^3 (2x^2 + 5x)dx = \lim_{h \to 0} h[f(1) + f(1+h) + f(1+2h) + ... + f(1+\overline{n-1}h)]$

where $f(x) = 2x^2 + 5x$ and $h = \frac{2}{n}$ or $nh = 2.$ 1

$f(1) = 7$

$f(1+h) = 2(1+h)^2 + 5(1+h) = 7 + 9h + 2^2$

$f(1+2h) = 2(1+2h)^2 + 5(1+2h) = 7 + 18h + 22^2 h^2$ 2

$f(1+3h) = 2(1+3h)^2 + 5(1+3h) = 7 + 27h + 2.3^2 h^2$

....

$f(1+(n-1)h) = 7 + 9(n-1)h + 2.(n-1)^2 h^2$

$\therefore \quad I = \lim_{h \to 0} h\left[7n + 9h\frac{n(n-1)}{2} + 2h^2.\frac{n(n-1)(2n-1)}{6}\right]$ 1

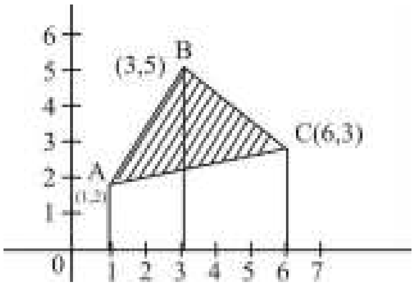
$= \lim_{h \to 0}\left[7nh + \frac{9}{2}nh(nh-h) + \frac{1}{3}nh(nh-h)(2nh-h)\right]$ 1

$= 14 + 18 + \frac{16}{3} = \frac{112}{3}$ 1

27. Let AB be $3x - 2y + 1 = 0$, BC be $2x + 3y - 21 = 0$ and AC be $x - 5y + 9 = 0$ correct figure : 1

Solving to get A(1, 2), B (3, 5) and C(6, 3) 1½

area of $(\Delta ABC) = \frac{1}{2}\int_1^3 (3x+1)dx + \frac{1}{3}\int_3^6 (21-2x)dx - \frac{1}{5}\int_1^6 (x+9)dx$ 1



$$= \left.\frac{1}{12}(3x+1)^2\right]_1^3 + \left.\frac{(21-2x)^2}{-12}\right]_3^6 - \left.\frac{(x+9)^2}{10}\right]_1^6 \qquad 1½$$

$$= 7 + 12 - \frac{25}{2} \qquad ½$$

$$= \frac{13}{2} \text{ sq. U.} \qquad ½$$

28. Surface area $A = 2\pi rh + 2\pi r^2$ (Given) ½

$$\Rightarrow \quad h = \frac{A - 2\pi r^2}{2\pi r} \qquad \text{...(1)} \qquad 1$$

$$V = \pi r^2 h = \pi r^2 \left(\frac{A - 2\pi r^2}{2\pi r}\right) \qquad 1$$

$$= \frac{1}{2}.[Ar - 2\pi r^3] \qquad ½$$

$$\frac{dv}{dr} = \frac{1}{2}[A - 6\pi r^2] \qquad 1$$

$$\frac{dv}{dr} = 0 = \Rightarrow \; 6\pi r^2 = A = 2\pi rh + 2\pi r^2 \qquad 1$$

$$\Rightarrow \; 4\pi r^2 = 2\pi rh \; \Rightarrow h = 2r = \text{diameter} \qquad 1$$

$$\frac{d^2v}{dr^2} = \frac{1}{2}[-12\pi r] < 0 \therefore \; h = 2r \text{ will give max. volume.} \qquad 1$$

29. Given equations can be written as

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \\ 12 \end{pmatrix} \text{ or } AX = B \qquad 1$$

$a_{11} = 7$, $a_{12} = -19$ $a_{13} = -11$
$a_{21} = 1$, $a_{22} = -1$ $a_{23} = -1$
$a_{31} = -3$, $a_{32} = 11$ $a_{33} = 72$

$$\Rightarrow \quad A^{-1} = \frac{1}{4}\begin{pmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{pmatrix} \qquad ½$$

$$\therefore \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{pmatrix}\begin{pmatrix} 7 \\ -5 \\ 12 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$\Rightarrow \quad x = 2, y = 1, z = 3.$ 1½

OR

Let $A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ $\therefore$ Writing $\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \mathbf{A}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 1

$c_1 \leftrightarrow c_2 \Rightarrow$ $\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 1 \end{pmatrix} = A\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ½

$c_2 \to c_2 + c_1$
$c_3 \to c_3 - 2c_1$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & -1 \\ 1 & 4 & -1 \end{pmatrix} = A\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ 1

$c_1 \to c_1 + 2c_3$
$c_2 \to c_2 + 2c_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 2 & -1 \end{pmatrix} = A\begin{pmatrix} 0 & 1 & 0 \\ -3 & -3 & -2 \\ 2 & 2 & 1 \end{pmatrix}$ ½

$c_3 \to c_3 + -c_2$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix} = A\begin{pmatrix} 0 & 1 & 1 \\ -3 & -3 & -5 \\ 2 & 2 & 3 \end{pmatrix}$ ½

$c_1 \to c_1 + c_3$
$c_2 \to c_2 + 2c_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = A\begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$ 1

$\Rightarrow$ $A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$ 1